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Mathematical Modelling of Syphilis Transmission Dynamics: Impacts of Mass Media Report, Risky Sexual Behavior and Treatment

Gasper Godson Mwanga
Department of Physics, Mathematics and Informatics, Dar es Salaam University College of Education, University of Dar es Salaam, P. O. Box 2329, Dar es Salaam Tanzania
E-mail: mwanga.gasper@gmail.com
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Abstract

Syphilis is one of the deadly sexually-transmitted diseases. This paper studied the impacts of sexual behavior, mass media report and treatment of infected individuals on the dynamics of syphilis transmission. The analyses of the full model and corresponding two sub-models are presented. The disease free equilibrium of the model is both locally and globally asymptotic stable when the associated reproduction number is less than unity. Analysis of the Reproduction number shows that it is not possible to control syphilis disease transmission if the rate of individuals practicing risky sexual behavior is high. Furthermore, the treatment of late (latent and tertiary) syphilis infection is beneficial to the infected individuals, but has no impact in the reproduction number. This study suggests that the effective control strategy of syphilis must focus on lowering the proportion of population practicing risky sexual behavior and applying higher treatment rates for early syphilis infections. Furthermore, the media items should address the issues regarding safe sexual behavior.

Keywords: Syphilis, sexually transmitted infection, Risky sexual behavior, Mass media.

Introduction

The world is experiencing the resurgence of the number of syphilis infection cases in many countries around the world (Halatoko et al. 2017). Syphilis infections are endemic in low and middle-income countries, while high-income countries are experiencing high syphilis burdens for groups of individuals who practice risky sexual behavior (Chico et al. 2021, Fenton et al. 2008, Halatoko et al. 2017). For example, the prevalence of syphilis infections among Men who have Sex with Men (MSM) across 77 countries is 7.5% compared to 0.5% among men in general population during the years 2000 to 2020 (Chico et al. 2021). Despite this fact, there are no mathematical models that attempt to model syphilis transmissions by stratifying sexually active population into risky sexual behavior and risky sexual behavior naïve group. According to the World Health Organization (WHO), there are more than one million sexually transmitted infections (STI) new cases every day. It is estimated that there are about 376 million new infection cases each year with one of four STIs: chlamydia (127 million), gonorrhea (87 million), syphilis (6.3 million) and trichomoniasis (156 million) (Rowley et al. 2019). In the year 2006, an estimate of 98,8000 pregnant women were infected with syphilis, resulting to over 350,000 stillbirths, giving birth to infants with congenital syphilis and deaths (Stoltey and Cohen 2015).

Syphilis is caused by bacteria called *Treponema pallidum* transmitted by penetration of these bacteria when come in contact with syphilis sore during vaginal, anal, or oral sex (Stoltey and Cohen 2015). Syphilis can also be prenatally transmitted from an infected mother to her unborn baby and through blood transfusion (Stoltey and Cohen 2015). Despite of the devastating health impact, syphilis infection can easily be cured by the administration of a single dose of *Benzanthine penicillin G, 2.4 mU* (Saad-Roy et al. 2016, Wagenlehner et al. 2016). Syphilis infections are commonly characterized into five distinct stages; exposed, primary infections, secondary infections, latent and tertiary infections. For the review of the clinical, epidemiologic, and biologic features of the syphilis, the reader is referred to Stoltey and Cohen (2015).

Syphilis like any other STIs although it can infect all sexually active individuals, but the chances of transmission are even higher for the individuals with risky sexual behaviors (For example see Aalsma et al. 2010, Kakchapati et al. 2017). The risky behavior in this paper refers to any individuals' behavior that leads to recklessly involvement in an unsafe sexual intercourse. Risky sexual behaviors which lead to the transmission of syphilis include, men to men sex (Saad-Roy et al. 2016); anal sex (Malhotra 2008, Mansergh et al. 2008); multiple sex partners (Wilson and Sathiyasusuman 2015); sexual intercourse without wearing condom (Todd et al. 2010); engaging in sex work (Kakchapati et al. 2017), and use of alcohol and drugs (Mansergh et al. 2008).

Mathematical models have been used to investigate the behavioral impacts on the spread of STIs. For example, the effects of heavy alcohol consumption on the transmission of gonorrhea (Bonyah et al. 2019); transmission of syphilis in Men who have Sex with Men (MSM) population (Saad-Roy et al. 2016); transmission of HIV in MSM population (Sorensen et al. 2012); effects of alcohol consumption on the progression and transmission of HIV (Ongodia and Mwanga 2020), HIV transmission on heterosexual, intravenous drug uses (IVDU) (Grassly et al. 2003), just to mention but a few. Furthermore, recently there is an increase in the use of mass media reports in the modelling of the dynamics of infectious disease. The media reports can induce the individual behavior change when reported correctly and hence positively improve public health. Huo et al. (2017) developed a social epidemic model to depict alcoholism with media coverage where they observed that media coverage is effective measure to quit drinking. Zhou et al. (2019) developed mathematical model for infectious diseases with a separate component of media coverage on the intensity of infections. They observed the low number of infection cases under the optimal reporting intensity.

Although there are research papers on the mathematical modelling of syphilis, for example, Milner and Zhao (2010), Nwankwo and Okuonghae (2018), Saad-Roy et al. (2016), and Tuite (2017), to the best of my knowledge, no any researcher had developed mathematical model of syphilis transmission with general risky sexual behavior individuals and impacts of the mass media coverage. This stratification of the population (behavioral risky and risky naïve) is of significance since various interventions advocated by the World Health Organization, managed to lower the rate of syphilis infections, but research shows the rates of syphilis infections continue to increase among select populations (World Health Organization 2016). In this paper, the effects of mass media report on the spread of syphilis are incorporated by considering the intensity of disease outbreaks.

Materials and Methods

Model formulation

A deterministic compartmental model where the temporal immunity gained upon *B. penicillin* treatment is ignored and hence the treated individual returns directly to susceptible class is developed. This assumption agrees with the finding that humans can still be infected with syphilis even after successful treatment (Stoltey and Cohen 2015). For simplicity in the modelling, since the primary role is to study the effects of risky sexual behavior and impacts of mass media reports on the transmissions of syphilis, the primary and secondary infection stages are grouped in to a single stage called, infectious class (I). Furthermore, the latent and tertiary infection classes are combined to form a late stage infections but not infectious (X). This model assumes that the only means of syphilis transmission is through sexual practices, thus the syphilis transmissions through infected mothers to her unborn babies (that is, vertical transmissions) and through blood transfusion are ignored. The sexually active individuals are stratified into two major groups, those with risky sexual behaviors (subscript p) and those who are not (subscript n). The individuals in each group are further divided into Susceptible S (those who are at risk of contracting infections), Exposed E (those who contracted infections but not infectious), Infectious I (those who are symptomatic and infectious) and late stage infected individuals X. Therefore, the total sexually active human population is

$$N_H = S_n + S_p + E_p + E_n + I_p + I_n + X_p + X_n$$

The mass media reports are incorporated by a separate compartment M in which the media influence the human behaviors on the contact rate, which lead to the infections. The sexually active individuals are recruited at maturity rate Λ, of these a proportional ρ engage in risky sexual behaviors. μ is the natural mortality, δ is the disease induced death rate for late stage infected individuals, β is the probability of disease transmission per sexual contact between infectious and susceptible individuals. The susceptible individuals with risky sexual behaviors are considered to have an increased contact rate to the infectious individuals at the rate $\sigma > 1$. τ_1 and τ_2 are the treatment rates for individuals in the infectious and late infected classes, respectively, $\frac{1}{\eta}$ is the incubation period of syphilis, $\frac{1}{\gamma}$ is the duration of infectious period of infected individual if remain untreated, ω and ϕ are the rates of behavioral changes from risky naïve group to risky sexual behavior group and vice versa, respectively. Like in Zhou et al. (2019), the propagation of information depends on the number of newly infected individuals $\left(\eta\left[E_n + E_p\right]\right)$ and ν is the reporting rate of newly observed infected cases. It is assumed that the media spontaneously disappear at a rate ε. $f\left(I_n, I_p, M\right)$ is the modification of syphilis contact rate which is induced by the media effects. The flow diagram of the model is represented in Figure 1.

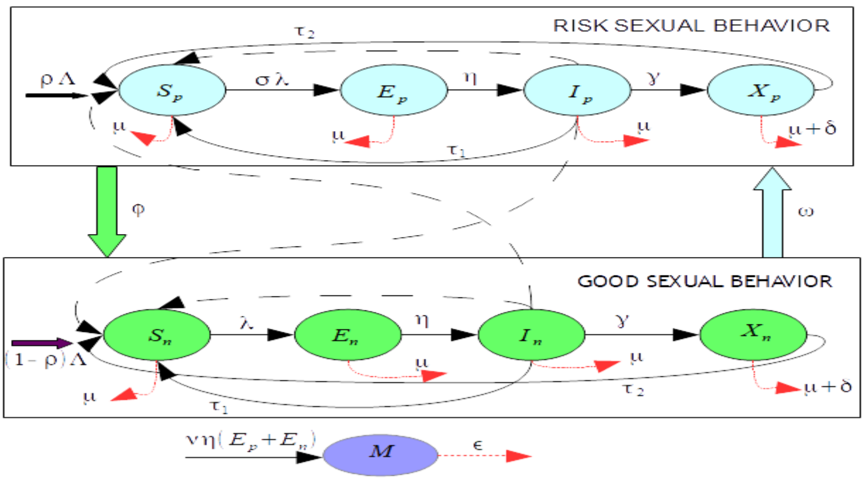

Figure 1: The Flux diagram of syphilis transmission stratifying the human population into risky sexual behavior and risky naïve groups. Solid continuous lines represent transfer from one compartment to another, dash lines represent infections and dotted red lines represent mortality or disappearance rates. The solid thick arrow shows the recruitment rate into the susceptible classes and behavioral changes between the two groups.

The dynamics of the model changing with time as a system of nonlinear ordinary differential equations is formulated as given in equation (1).

$$\frac{dS_n}{dt} = (1-\rho)\Lambda + \tau_1 I_n + \tau_2 X_n + \phi S_p - f\left(I_n, I_p, M\right)\beta\left(I_p + I_n\right)S_n - (\mu+\omega)S_n$$

$$\frac{dE_n}{dt} = f\left(I_n, I_p, M\right)\beta\left(I_p + I_n\right)S_n + \phi E_p - (\mu+\omega+\eta)E_n$$

$$\frac{dI_n}{dt} = \eta E_n + \phi I_p - (\gamma+\mu+\omega+\tau_1)I_n$$

$$\frac{dX_n}{dt} = \gamma I_n + \phi X_p - (\delta+\mu+\omega+\tau_2)X_n$$

$$\frac{dS_p}{dt} = \rho\Lambda + \tau_1 I_p + \tau_2 X_p + \omega S_n - f\left(I_n, I_p, M\right)\sigma\beta\left(I_p + I_n\right)S_p - (\mu+\phi)S_p$$

$$\frac{dE_p}{dt} = f\left(I_n, I_p, M\right)\sigma\beta\left(I_p + I_n\right)S_p + \omega E_n - (\mu+\phi+\eta)E_p$$

$$\frac{dI_p}{dt} = \eta E_p + \omega I_n - (\gamma+\mu+\phi+\tau_1)I_p$$

$$\frac{dX_p}{dt} = \gamma I_p + \omega X_n - (\delta+\mu+\phi+\tau_2)X_p$$

$$\frac{dM}{dt} = \nu\eta\left(E_n + E_p\right) - \varepsilon M \tag{1}$$

$$f(I_p, I_n, M) = \frac{1}{1+\alpha_1(I_p + I_n) + \alpha_2 M} \tag{2}$$

The media function $f(I_p, I_n, M)$ is chosen such that is the decreasing function with respect to the number of infected individuals I_p and I_n, and the media intensity M. This choice assumes that the human behavior towards infections changes with the number of infections and intensity of media reports on infections (Zhou et al. 2019). The basic properties required to be satisfied by any epidemiologic mathematical model based on ordinary differential equations is studied in the next section.

Invariant region of the model

Lemma 1.1: All feasible regions Ω *defined by*

$$\Omega = \left\{\left(S_n(t), E_n(t), I_n(t), X_n(t), S_p(t), E_p(t), I_p(t), X_p(t), M(t)\right) \in \mathbb{R}^9_+ : N_H(t) \le \frac{\Lambda}{\mu};\ M(t) \le \frac{\nu\eta\Lambda}{\varepsilon\mu}\right\}$$

with initial conditions $S_n(0)\ge 0,\ E_n(0)\ge 0,\ I_n(0)\ge 0,\ X_n(0)\ge 0,\ S_p(0)\ge 0,$ $E_p(0)\ge 0,\ I_p(0)\ge 0,\ X_p(0)\ge 0$ $, M(0)\ge 0$ *are positively invariant for system (1).*

Proof: Since $N_H'(t) = S_n'(t) + E_n'(t) + I_n'(t) + X_n'(t) + S_p'(t) + E_p'(t) + I_p'(t) + X_p'(t)$

$$= \Lambda - \mu N_H - \delta\left(X_n + X_p\right)$$

$$\frac{dN_H}{dt} + \mu N_H \le \Lambda \tag{3}$$

Integrating both sides of the inequality (3) results to

$$0 \le N_H(t) \le \frac{\Lambda}{\mu}\left(1-e^{-\mu t}\right)+N_H(0)e^{-\mu t}$$

Thus $\lim\sup_{t\to\infty} N_H(t) \le \frac{\Lambda}{\mu}.$

From the last equation in the system (1)

$$\frac{dM}{dt}=\nu\eta\left(E_n+E_p\right)-\varepsilon M$$

Since $0 \le E_n+E_p \le N_H \le \frac{\Lambda}{\mu}$

It results to $\lim\sup_{t\to\infty} M(t) \le \frac{\nu\eta\Lambda}{\varepsilon\mu}.$

Hence, the feasible region of system (1) given by Ω is positively invariant with respect to system (1).

Positivity of solution

For system (1) to ensure that its solution with positive initial conditions remains positive for all $t>0$, it is necessary to prove that all the system variables are positive, therefore we have the following Lemma:

Lemma 1.2: If $S_n(0)\ge 0,\ E_n(0)\ge 0,\ I_n(0)\ge 0,\ X_n(0)\ge 0,\ S_p(0)\ge 0,\ E_p(0)\ge 0,\ I_p(0)\ge 0,$ $X_p(0)\ge 0$ *and* $M(0)\ge 0$; *then the solutions* $S_n(t), E_n(t), I_n(t), X_n(t), S_p(t), E_p(t),$ $I_p(t), X_p(t)$ *and* $M(t)$ *of the system* (1) *are positive for all* $t\ge 0.$

Proof: Given the positive initial conditions, it is easy to prove that the solutions of system (1) are positive; if not, it is proved by contradiction that there exists a first time

$$t_1 : S_n(t_1)=0,\ S_n'(t_1)<0,\ S_n(t)>0,\ E_n(t)\ge 0,\ \cdots,\ M(t)\ge 0 \text{ for } 0<t<t_1$$

In that case the first equation in the system (1) results to

$$S_n'(t_1)=(1-\rho)\Lambda+\tau_1 I_n(t_1)+\tau_2 X_n(t_1)+\varphi S_p(t_1)>0;\ \ \rho<1$$

which is a contradiction meaning that $S_n(t)\ge 0,\ \forall t>0.$

Or there exists a $t_2 : E_n(t_2)=0,\ E_n'(t_2)<0,\ S_n(t)>0,\ E_n(t)\ge 0,\ \cdots,\ M(t)\ge 0$ for $0<t<t_2$

In that case, the second equation in the system (1) results to

$$E_n'(t_2)=\varphi E_p(t_2)+f\left(I_n, I_p, \mathrm{M}\right)\beta\left[I_p(t_2)+I_n(t_2)\right]S_n(t_2)\ >\ 0$$

which is a contradiction meaning that $E_n(t)\ge 0,\ \forall t>0.$

Similarly choosing $t_3,\ t_4,\ t_5,\ t_6,\ t_7,\ t_8$ correspondingly it can be shown that

$$I_n(t), X_n(t), \mathrm{S}_p(t), E_p(t), I_p(t), X_p(t)\ge 0,\ \forall t>0.$$

Or there exists a

$t_9 : M(t_9)=0,\ M'(t_9)<0,\ S_n(t)>0,\ E_n(t)\ge 0,\ \cdots,\ M(t)\ge 0$ for $0<t<t_9$

In that case, the ninth equation in the system (1) results to

$$M'(t_9) = \nu\eta\left[E_n(t_9) + E_p(t_9)\right] > 0$$

which is a contradiction meaning that $M(t) \geq 0, \ \forall t > 0.$

Thus, the solution $S_n(t), E_n(t), I_n(t), X_n(t), S_p(t), E_p(t), I_p(t), X_p(t)$ and $M(t)$ of the system (1) are positive for all $t \geq 0.$

Disease free equilibrium and the reproductive number

Disease free equilibrium (DFE) is the point at which the population is free from infections. Therefore, the population is entirely comprised of only susceptible individuals who fall in two behavioral categories; those with risky sexual behaviors and those who are not. This is obtained by finding the stationary point of the system (1) when

$$E_n = I_n = X_n = E_p = I_p = X_p = M = 0.$$

Denoting $Y^0 = \left\{S_n^0, E_n^0, I_n^0, X_n^0, S_p^0, E_p^0, I_p^0, X_p^0, M^0\right\}$ be the disease free equilibrium point of the system (1) the following result is obtained:

$$Y^0 = \left\{\frac{\Lambda\left[\mu(1-\rho)+\phi\right]}{\mu(\omega+\mu+\phi)}, 0, 0, 0, \frac{\Lambda(\omega+\rho\mu)}{\mu(\omega+\mu+\phi)}, 0, 0, 0, 0\right\} \tag{4}$$

Using the Next Generation Method developed by Van Den Driessche and Watmough (2002), the basic reproductive number, for the model system (1) is computed. The basic reproduction number R_0 is the expected number of secondary infections produced by a single infected individual during its entire period of infectiousness in a completely susceptible population. In this model, there are two routes for infections, that is, through E_p or E_n. To study the structure of each sub-model from these two transmission routes, briefly their analyses are presented hereunder.

Case 1: Entire population practice safe sexual behavior

The population dynamics for sexually active human who practice safe sexual behavior is obtained when, $S_p = E_p = I_p = X_p = 0,$ and $\phi = \omega = \rho = 0.$ Therefore, system (1) reduces to

$$\begin{aligned}
\frac{dS_n}{dt} &= \Lambda + \tau_1 I_n + \tau_2 X_n - f(I_n, M)\beta I_n S_n - \mu S_n \\
\frac{dE_n}{dt} &= f(I_n, M)\beta I_n S_n - (\mu+\eta)E_n \\
\frac{dI_n}{dt} &= \eta E_n - (\gamma+\mu+\tau_1)I_n \\
\frac{dX_n}{dt} &= \gamma I_n - (\delta+\mu+\tau_2)X_n \\
\frac{dM}{dt} &= \nu\eta E_n - \varepsilon M
\end{aligned} \tag{5}$$

where $f(I_n, M) = \dfrac{1}{1+\alpha_1 I_n + \alpha_2 M}.$

The disease-free equilibrium of the model system (5) is given by $Y_n^0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right)$. The basic reproduction number of model system (5) is thus given by;

$$R_0^n = \frac{\beta\eta\Lambda}{\mu(\mu+\eta)(\mu+\gamma+\tau_1)} \tag{6}$$

To achieve $R_0^n < 1$, the treatment rate (τ_1) can either be increased or reduce the probability of disease transmission per sexual contact between infectious and susceptible individuals (β).

Theorem 1.1: The disease-free equilibrium point of the model system (5) *is locally asymptotically stable if* $R_0^n < 1$.

Proof: The proof involves evaluating the Jacobian Matrix of system (5) at Y_n^0 which gives

$$J\left(Y_n^0\right) = \begin{pmatrix} -\mu & 0 & \tau_1 - S_n^0 & \tau_2 & 0 \\ 0 & -a_1 & S_n^0 & 0 & 0 \\ 0 & \eta & -a_2 & 0 & 0 \\ 0 & 0 & \gamma & -a_3 & 0 \\ 0 & \nu\eta & 0 & 0 & -\varepsilon \end{pmatrix}$$

where $a_1 = \mu + \eta$, $a_2 = \gamma + \mu + \tau_1$ and $a_3 = \delta + \mu + \tau_2$.

The Jacobian of $J\left(Y_n^0\right)$ is;

$$p(\lambda) = (\lambda+\mu)(\lambda+\varepsilon)(\lambda+a_3)\left[\lambda^2 + b_1\lambda + b_2\right]$$

Since all roots of $p(\lambda)$ are negative, therefore, according to Routh-Hurwitz criteria the disease free equilibrium Y_n^0 is locally asymptotically stable. Note $b_1 = a_1 a_2 > 0$ and $b_2 = a_1 a_2 - \eta\frac{\Lambda}{\mu} > 0$

Case 2: Entire population practice risky sexual behavior

The risky sexual behavior model is obtained when, $S_n = E_n = I_n = X_n = 0$, $\phi = \omega = 0$ and $\rho = 1$. Therefore, system (1) reduces to

$$\begin{aligned}
\frac{dS_p}{dt} &= \Lambda + \tau_1 I_p + \tau_2 X_p - f\left(I_p, M\right)\sigma\beta I_p S_p - \mu S_p \\
\frac{dE_p}{dt} &= f\left(I_p, M\right)\sigma\beta I_p S_p - (\mu+\eta)E_p \\
\frac{dI_p}{dt} &= \eta E_p - (\gamma+\mu+\tau_1) I_p \\
\frac{dX_p}{dt} &= \gamma I_p - (\delta+\mu+\tau_2) X_p \\
\frac{dM}{dt} &= \nu\eta E_p - \varepsilon M
\end{aligned} \tag{7}$$

where $f(I_p, M) = \dfrac{1}{1+\alpha_1 I_p + \alpha_2 M}$.

The disease-free equilibrium of the model system (7) is given by $Y_p^0 = \left(\dfrac{\Lambda}{\mu}, 0, 0, 0, 0\right)$. Using the Next Generation Method, the basic reproduction number of model system (7) is thus given by;

$$R_0^p = \frac{\beta\eta\sigma\Lambda}{\mu(\mu+\eta)(\mu+\gamma+\tau_1)} \tag{8}$$

To achieve $R_0^p < 1$, the treatment rate (τ_1) can either increased and/or reduce the probability of disease transmission per sexual contact between infectious and susceptible individuals (β) and the contact rate (σ).

Theorem 1.2: The disease-free equilibrium point of the model system (7) *is locally asymptotically stable if* $R_0^p < 1$.

Proof: Similarly evaluating the Jacobian Matrix of system (7) at Y_p^0 gives

$$J\left(Y_p^0\right) = \begin{pmatrix} -\mu & 0 & \tau_1 - S_p^0 & \tau_2 & 0 \\ 0 & -a_1 & S_p^0 & 0 & 0 \\ 0 & \eta & -a_2 & 0 & 0 \\ 0 & 0 & \gamma & -a_3 & 0 \\ 0 & \nu\eta & 0 & 0 & -\varepsilon \end{pmatrix}$$

where $a_1 = \mu + \eta$, $a_2 = \gamma + \mu + \tau_1$ and $a_3 = \delta + \mu + \tau_2$.

The Jacobian of $J\left(Y_p^0\right)$ is;

$$l(\lambda) = (\lambda+\mu)(\lambda+\varepsilon)(\lambda+a_3)\left[\lambda^2 + b_1\lambda + b_2\right]$$

Since all roots of $l(\lambda)$ are negative, therefore, according to Routh-Hurwitz criteria the disease free equilibrium Y_p^0 is locally asymptotically stable. Note $b_1 = a_1 a_2 > 0$ and $b_2 = a_1 a_2 - \eta\dfrac{\Lambda}{\mu} > 0$

Conclusively, it can be seen from equations (6) and (8) that $R_0^p = \sigma R_0^n$, that is, the reproduction number for the population which is entirely practicing risky sexual behavior exceeds that of the population which is entirely practicing safe sexual behavior by the factor $\sigma > 1$. For example, if $\sigma = 10$, and $R_0^n = 0.2 < 1$ then $R_0^p = 2 > 1$ therefore, the disease will die out in the population which practice safe sexual behavior while the disease will become epidemic in the population which practice risky sexual behavior.

Case 3: Co-existence of individuals with safe and risky sexual behaviors

The reproductive number, R_0 for the full model system (1) is computed by first obtaining the matrices F of the transmission terms and the matrix V of the transition terms calculated at Y^0 as follows:

$$F=\begin{pmatrix} 0 & \beta S_n^0 & 0 & 0 & \beta S_n^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta\sigma S_p^0 & 0 & 0 & \beta\sigma S_p^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } V=\begin{pmatrix} c_1 & 0 & 0 & -\phi & 0 & 0 \\ -\eta & c_2 & 0 & 0 & -\phi & 0 \\ 0 & -\gamma & c_3 & 0 & 0 & -\phi \\ -\omega & 0 & 0 & c_4 & 0 & 0 \\ 0 & -\omega & 0 & -\eta & c_5 & 0 \\ 0 & 0 & -\omega & 0 & -\gamma & c_6 \end{pmatrix}$$

R_0 is computed as the spectral radius of the matrix FV^{-1} gives equation (9).

$$R_0=\frac{\beta\eta\left[S_p^0\left(\sigma g\left(\phi\omega+c_1c_2\right)+\sigma\phi c_5\right)+S_n^0\left(g\left(\omega c_2+c_4\right)+\phi\omega+c_4c_5\right)\right]}{\phi\omega\left(\phi\omega-c_1c_4-c_2c_5\right)+c_1c_2c_4c_5} \tag{9}$$

where,

$c_1=\mu+\omega+\eta,\ c_2=\gamma+\mu+\omega+\tau_1,\ c_3=\delta+\mu+\omega+\tau_2,\ c_4=\mu+\phi+\eta,\ c_5=\gamma+\mu+\phi+\tau_1$

$c_6=\delta+\mu+\phi+\tau_2$ and $S_n^0=\dfrac{\Lambda\left[\mu(1-\rho)+\phi\right]}{\mu\left(\omega+\mu+\phi\right)},\ S_p^0=\dfrac{\Lambda\left(\omega+\rho\mu\right)}{\mu\left(\omega+\mu+\phi\right)}.$

Analysis of the reproduction number, R_0

To determine the effect of the proportional of individuals who change their behavior to practice risky sexual behavior, ω and treatment parameter, τ_1 on the reproduction number R_0 the following limits (Saad-Roy et al. 2016), are considered:

$$\lim_{\omega\to\infty} R_0=\infty. \tag{10}$$

$$\lim_{\omega\to 0} R_0=T, \tag{11}$$

where,

$$T=\frac{\Lambda\rho\sigma}{\mu+\phi}\left[\frac{\phi}{c_4}\left(\frac{1}{(\gamma+\mu+\phi+\tau_1)(\gamma+\mu+\tau_1)}+\frac{1}{(\mu+\eta)(\gamma+\mu+\tau_1)}+\frac{1}{(\gamma+\mu+\phi+\tau_1)}\right)\right]+\frac{\Lambda\left[\mu(1-\rho)+\phi\right]}{\mu(\mu+\phi)(\mu+\eta)(\gamma+\mu+\tau_1)}$$

and therefore,

$$\lim_{\tau_1\to\infty} T=0. \tag{12}$$

Therefore, it follows from (10) that when a large proportion of sexually active individuals engage in risky sexual behavior, it is not possible to control the syphilis infections in the community since the limit in (10) approaches infinity. However, from (11) when the rate of individuals engaging in risky sexual behavior near zero, near total syphilis eradication is possible if high treatment rate to the syphilis infected individuals during the early stage is concurrently applied, since the limit in (12) approaches zero.

Computing the forward sensitivity index of R_0 with respect to ω, ϕ, τ_1 and τ_2 using the parameter values given in Table 1, further reveals the effects of these parameters on the control of syphilis transmission in the population. The following results are obtained:

$$S_{\omega} = \frac{\partial R_0}{\partial \omega}\frac{\omega}{R_0} = 0.6269,\; S_{\phi} = \frac{\partial R_0}{\partial \phi}\frac{\phi}{R_0} = -0.6422,\; S_{\tau_1} = \frac{\partial R_0}{\partial \tau_1}\frac{\tau_1}{R_0} = -0.5061 \text{ and } S_{\tau_2} = \frac{\partial R_0}{\partial \tau_2}\frac{\tau_2}{R_0} = 0.$$

Since $S_{\omega} > 0$, then to control the rate of syphilis infections, the rate of individuals changing behavior from behavioral risky naïve to risky sexual behavior need to be reduced. On the other hand $S_{\phi}, S_{\tau_1} < 0$ implying that to have an effective control strategy of syphilis infections, the rates of individuals who change behavior from risky sexual behavior to behavioral risky naïve are required to be increased and/or increase the treatment rate for early syphilis infections. It is worth noting that $S_{\tau_2} = 0$, thus treatment of the late syphilis infections has no significant impacts on the rates of syphilis transmissions.

Local stability of the disease free equilibrium

Theorem 1.3: The disease-free equilibrium point of the model system (1) *is locally asymptotically stable if* $R_0 < 1$.

Proof: The proof of this theorem can be done in similar ways as in Theorem 1.1 and Theorem 1.2.

Global stability of the disease free equilibrium

Using the method of Castillo-Chavez et al. (2002), the set of equation in the model system (1) is re-written in the form

$$\frac{dY}{dt} = F(Y, Z)$$
$$\frac{dZ}{dt} = G(Y, Z)$$

with $G(Y,0) = 0$. $Y \in \mathbb{R}^2$ denoting uninfected compartments (S_n, S_p) and $Z \in \mathbb{R}^7$ denoting infected compartments $(E_n, I_n, X_n, E_p, I_p, X_p, M)$. $Y^0 = (Y^*, 0)$ is the disease free equilibrium given in equation (4). The two conditions (H1) and (H2) below must be satisfied for global stability.

H1: For $\frac{dY}{dt} = F(Y,0)$; $Y*$ is globaly asymptotically stable.

H2: $G(Y,Z) = AZ - \hat{G}(Y,Z)$, $\hat{G}(Y,Z) \geq 0$ for $(Y,Z) \in \Omega$

Where $A = D_Z G(Y^*, 0)$ is and M matrix (the off-diagonal elements of A are nonnegative) and Ω is the feasible region where the model is biologically meaningful. If conditions (H1) and (H2) are satisfied, then the following theorem holds:

Theorem 1.4: (Castillo-Chavez et al. 2002)*: The fixed point* $Y^0 = (Y^*, 0)$ *is globally stable equilibrium of model given by equation* (4) *provided that* $R_0 < 1$.

Proof:

$$F(Y,0)=\begin{pmatrix}(1-\rho)\Lambda+\phi S_p-(\mu+\omega)S_n\\ \rho\Lambda+\omega S_n-(\mu+\phi)S_p\end{pmatrix},\ A=\begin{pmatrix}-c_1 & \beta S_n^0 & 0 & \phi & \beta S_n^0 & 0 & 0\\ \eta & -c_2 & 0 & 0 & \phi & 0 & 0\\ 0 & \gamma & -c_3 & 0 & 0 & \phi & 0\\ \omega & \sigma\beta S_p^0 & 0 & -c_4 & \sigma\beta S_p^0 & 0 & 0\\ 0 & \omega & 0 & \eta & -c_5 & 0 & 0\\ 0 & 0 & \omega & 0 & \gamma & -c_6 & 0\\ \nu\eta & 0 & 0 & \nu\mu & 0 & 0 & -\varepsilon\end{pmatrix} \text{ and}$$

$$\hat{G}(Y,Z)=\begin{pmatrix}\hat{G}_1(Y,Z)\\ \hat{G}_2(Y,Z)\\ \hat{G}_3(Y,Z)\\ \hat{G}_4(Y,Z)\\ \hat{G}_5(Y,Z)\\ \hat{G}_6(Y,Z)\\ \hat{G}_7(Y,Z)\end{pmatrix}=\begin{pmatrix}\beta(I_p+I_n)\left(S_n^0-f(I_n,I_p,M)S_n\right)\\ 0\\ 0\\ \beta(I_p+I_n)\left(S_p^0-f(I_n,I_p,M)S_p\right)\\ 0\\ 0\\ 0\end{pmatrix}$$

Obviously Y^* is globally asymptotically stable of $\frac{dY}{dt}=F(Y,0)$. It is clear that A is the M-matrix and $\hat{G}(Y,Z)\geq 0$ (that is, $\hat{G}_1(Y,Z)\geq 0, \hat{G}_2(Y,Z)=0, \hat{G}_3(Y,Z)=0,\ \hat{G}_4(Y,Z)\geq 0,$ $\hat{G}_5(Y,Z)=0, \hat{G}_6(Y,Z)=0$ and $\hat{G}_7(Y,Z)=0$) since $f(I_n,I_p,M)\leq 1,$ $S_n\leq S_n^0$ and $S_p\leq S_p^0$ for $(Y,Z)\in\Omega$ and therefore, Y^0 is globally asymptotically stable.

Numerical Results and Discussion

For numerical simulation, the population statistics of Kinondoni District in Dar es salaam Tanzania as per 2012 National census (National Bureau of Statistics 2013) were used for the estimation of initial conditions of the model. This study considered the youth population between ages 20 to 29 years, whereby there were 449,710 youths in Kinondoni District. In this population, 43% of the youths were considered to practice risky sexual behavior (Fetene and Mekonnen 2018), thus $S_n^0=255885$ and $S_p^0=193825$. Previous studies have shown a wide range of syphilis infections, from 1.5% to 42.1% (Halatoko et al. 2017). Here 40% (77530) of the population with risky sexual behavior are assumed to be infected with syphilis, while only 1.5% (3839) of those who practice safe sexual behavior are assumed to have syphilis infections. Finally, it is further assumed that, of those infected individuals 50% are in exposed stage, 30% are in early infected stage, and the remaining 20% are in the late infected stage. Other model parameter values are as given in Table 1.

Table 1: Baseline parameter values as estimated from various sources in the literature all normalized to be per day

Parameters	Baseline (Range)	Source
Λ	13 (9.5 – 17.8)	(Hope Ifeyinwa 2020)
ρ	0.45	(Fetene and Mekonnen 2018)
τ_1, τ_2	0.36	(Garnett et al. 1996)
ω	0.431	Assumed
β	0.0353888	(Saad-Roy et al. 2016)
μ	0.0000589 (0.0000508 -0.0000589)	(Hope Ifeyinwa 2020)
ϕ	Variable (0 -1)	(Fetene and Mekonnen 2018)
η	1/46 (1/47 – 1/51)	(Garnett et al. 1996)
γ	1/3.6*30 (0.024 – 0.042)	(Garnett et al. 1996)
δ	0.000325 (0.000227 – 0.000414)	(Peterman and Kidd 2019)
σ	100	Assumed
ν, ε	0.01, 0.2535	(Zhou et al. 2019)

Figure 2 shows the effects of increasing the rates of individuals who practice risky sexual behavior. The figure clearly shows that the number of infected individuals in behavioral risky group increases as the population is inclined towards risky sexual behavior. This agrees with the mathematical analysis which shows that letting the value of ω to approach infinity, the value of R_0 approaches infinity too (10), and hence the disease can never be controlled. Therefore, a successful syphilis disease control strategy must include the control measures that lower the rates of individuals who practice risky sexual behaviors.

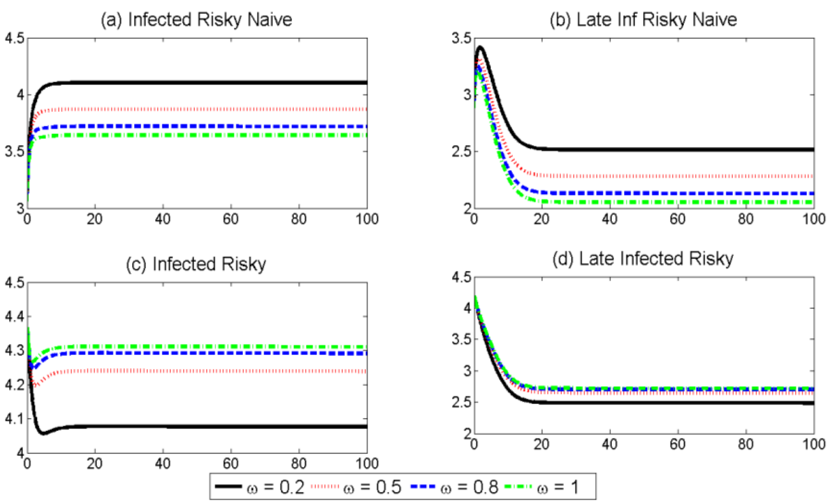

Figure 2: The comparison of state solutions for different rates of individuals changing from safe sexual behavior to risky sexual behavior. The vertical axis is in logarithm to base ten scale.

Figure 3 portrays the effects of changing the value of the media function. The figure shows that when there is no media function, the number of infected individuals remains slightly higher than the case with media function. In this model, the media function was modeled in such a way that it controls the incidence term by reducing the contact rate, but did not lead to the change of sexual practices (that is, did not affect parameter ϕ and ω). Therefore, successful media items need to include a component for changing sexual behavior parameters while lowering the infection rates.

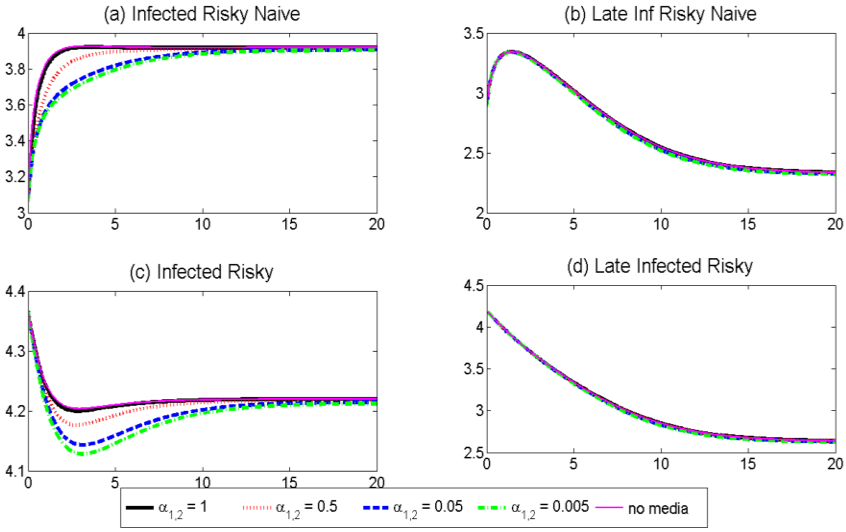

Figure 3: The comparison of state solutions of system (1) for different levels of media function. The vertical axis is in logarithm to base ten scale.

Figure 4 shows the effects of changing the treatment rates for the infected individuals. This figure shows the decline in the number of infected individuals with the increase in the values of the treatment rates. This agrees with the mathematical analysis in equation (12), which shows that with low rates of individual practicing risky sexual behavior, increasing the values of treatment rates leads to the decrease in the value of reproduction number. Similar observation was made by Saad-Roy et al. (2016). Therefore, higher treatment rates for syphilis infected individuals are required to control syphilis transmissions in the population.

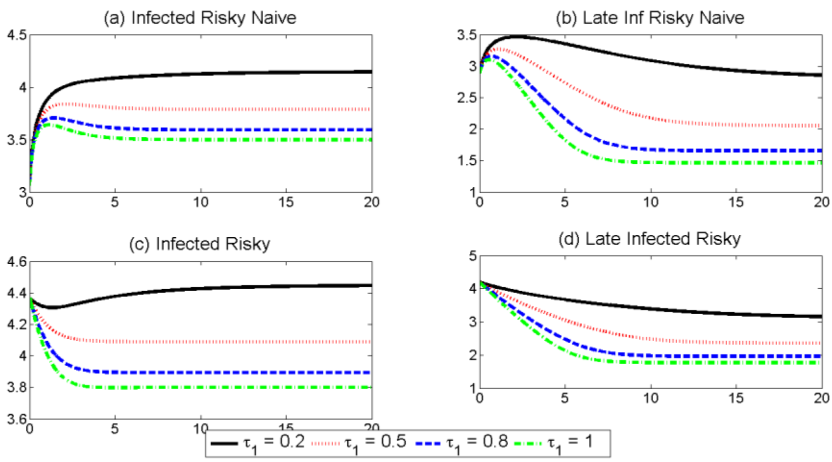

Figure 4: The comparison of state solutions of system (1) for different values of treatment rates $(\tau_1 = \tau_2)$. The vertical axis is in logarithm to base ten scale.

Conclusion

Syphilis is one of the deadly sexually-transmitted diseases. Like other literature, this paper made an attempt to study the impacts of sexual behavior and mass media reports to the dynamics of syphilis transmissions. The sexually active individuals are stratified into two major groups, those with risky sexual behaviors and those who practice safe sexual behaviors. This article defined the risky sexual behavior to refer to any individuals' behavior that leads to recklessly involvement in an unsafe sexual intercourse. The model system is both locally and globally stable when the basic reproduction number is less than a unity. Further analysis of the sub-models systems (entire population practicing safe sexual behaviors and entire population practicing risky sexual behaviors) reveals that are both locally stable when their corresponding basic reproduction number is less than unity. Comparison between the reproduction numbers of the two sub-models showed that the syphilis disease of the same outbreak level can die out if the entire population practices safe sexual behaviors, but become epidemic if the entire population practices risky sexual behaviors. This study shows that the disease cannot be controlled if the rate of individuals practicing risky sexual behavior is very high. Furthermore, the mass media items which target only on lowering the incidence term without changing the population sexual behavior parameters have little impacts to the dynamics of the syphilis transmissions. Nevertheless, the treatment of late (latent and tertiary) syphilis infections is beneficial to the infected individuals, but has no impact in the reproduction number. Analytical and numerical solutions demonstrate that there is a decline in the number of infected individuals with the increase in the values of the treatment rates. This study suggests that the effective control strategy of syphilis must focus on reducing the proportion of population who practice risky sexual behaviors and concurrently apply higher treatment rates for early stages of syphilis infections. Furthermore, the media functions must be addressed in such a way that they lead to the changes of sexual behaviors of the population (that is, to address the issue of positive sexual health).

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